On n-fold sum of a non-flat continuum

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(joint work with Taras Banakh and Wojciech Jabłoński)

We show that for a continuum $K \subset \mathbb{R}^n$ the sum K^{+n} of n copies of K has non-empty interior in \mathbb{R}^n if and only if K is not flat in the sense that the affine hull of K coincides with \mathbb{R}^n . Moreover, if K is locally connected and each non-empty open subset of K is not flat, then for any (analytic) non-meager subset $A \subset K$ the sum A^{+n} of n copies of A is not meager in \mathbb{R}^n (and then the sum A^{+2n} of 2n copies of the analytic set A has non-empty interior in \mathbb{R}^n and the set $(A - A)^{+n}$ is a neighborhood of zero in \mathbb{R}^n).

Our results refer to Ger's theorem [1] and also a result by Kallman and Simmons [2].

References

- R. Ger, *Thin sets and convex functions*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 21 (1973), 413–416.
- [2] R.R. Kallman, F.W. Simmons, A theorem on planar continua and an application to authomorphisms of the field of complex numbers, Topology Appl. 20 (1985), 251–255.